

Designing a solar powered Stirling heat engine based on multiple criteria: Maximized thermal efficiency and power



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ABSTRACT

A solar-powered high temperature differential Stirling engine was considered for optimization using multiple criteria. A thermal model was developed so that the output power and thermal efficiency of the solar Stirling system with finite rate of heat transfer, regenerative heat loss, conductive thermal bridging loss, finite regeneration process time and imperfect performance of the dish collector could be obtained. The output power and overall thermal efficiency were considered for simultaneous maximization. Multi-objective evolutionary algorithms (MOEAs) based on the NSGA-II algorithm were employed while the solar absorber temperature and the highest and lowest temperatures of the working fluid were considered the decision variables. The Pareto optimal frontier was obtained and a final optimal solution was also selected using various decision-making methods including the fuzzy Bellman–Zadeh, LINMAP and TOPSIS. It was found that multi-objective optimization could yield results with a relatively low deviation from the ideal solution in comparison to the conventional single objective approach. Furthermore, it was shown that, if the weight of thermal efficiency as one of the objective functions is considered to be greater than weight of the power objective, lower absorber temperature and low temperature ratio should be considered in the design of the Stirling engine.

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1. Introduction

Stirling cycle is one of the important primitive standard air cycles of heat engines [1,2]. Suitable efficiency and wide range of fuels that can be used for heating can be pointed out as some advantages of this type of engine [2–4]. The Stirling engine can theoretically be a very efficient engine to convert heat into mechanical work at the Carnot efficiency when the ideal regeneration and isothermal compression and expansion processes are considered. Thermal limit of the operation of the Stirling engine depends on working temperatures on the heater and cooler sides. In most instances, the engine operates with heater and cooler temperatures of 923 and 338 K, respectively [5]. Engine efficiency ranges from 30% to 40% coming from a typical temperature range of 923–1073 K and the normal operating speed range of 2000–4000 RPM [6–11]. Kongtragool and Wongwises [8] studied the influence of regenerator efficiency and dead volume on the work as well as efficiency of the machine. However, this study did not include heat transfers through the temperature difference at the heat source and sink.

The idea of coupling for solar concentrators to Stirling engines is a new technology which facilitates conversion of solar energy into electric power. In this regard, a dish collector with the parabolic arrangement of its mirrors is used to concentrate solar radiations on the focal point of the collector, in which heat absorber of the engine is located. Therefore, solar energy is collected and concentrated toward a parabola of mirrors.

In recent years, studying the optimum performance of energy systems with thermodynamic models has been an attractive option for researchers [12–34]. It is believed that the results obtained using the FTT lead to a more realistic design of the actual solar energy systems in comparison to the prior conventional thermodynamic equilibrium methods. Ladas and Ibrahim [28] defined a finite-time parameter as the ratio of working fluid contact time to the engine time constant which evolves heat transfer characteristics of the given design of Stirling engine. They conducted a numerical study and plotted variation of the power output versus finite-time parameter and change of the power output versus efficiency. Kaushik et al. studied effects of irreversibilities of the regenerator and heat transfer process in heat/sink sources [29]. Sieniutycz and von Spakovsky [32] generalized thermal exergy to the finite-time processes. Sahin and Kodal [33] introduced a new thermoeconomic performance analysis based on an objective function defined as the power output per unit total cost. Tili investigated effects of regenerating effectiveness and heat capacitance

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Nomenclature

A_{rec}	absorber (recuperate) area	X	vector of decision variables
A_{app}	aperture area	<i>Greek letter</i>	
C	concentration ratio	λ	ratio of volume during the regenerative processes (volumetric ratio)
C_V	molar specific heat capacity ($\text{J mol}^{-1} \text{K}^{-1}$)	η	thermal efficiency
d	deviation index	ε_R	effectiveness of the regenerator
d_{i+}	deviation or distance of i th solution from the ideal solution	ε	emissivity factor
d_{i-}	deviation or distance of i th solution from the non-ideal solution	δ	Stefan–Boltzman coefficient
h	heat transfer coefficient, (W K^{-1} or W K^{-4} or $\text{W m}^{-2} \text{K}^{-1}$)	<i>Subscripts</i>	
I	direct solar flux intensity (W m^{-2})	H	absorber (heater)
i	i th objective	HC	convection on the high temperature side heat exchanger
j	j th solution	HR	radiation on the high temperature side heat exchanger
n	mole number of the working fluid (mol)	L	heat sink
p	power (W)	m	entire solar-dish Stirling system
Q	heat transfer (J)	R	regenerator
R	universal gas constant ($\text{J mol}^{-1} \text{K}^{-1}$)	t	Stirling engine
T	temperature (K)	0	ambient condition, optics
W	work (J)	$4 - 1$	process states
t	cyclic period (s)		
k_0	conductive thermal bridge loss coefficient (W K^{-1})		
x	temperature ratio of the Stirling engine		

rate of external fluids at the heat source/sink on the maximum power and efficiency [34].

Yaqi et al. developed a mathematical model for the output power and overall thermal efficiency of the high temperature differential solar dish Stirling engine with the finite heat transfer and a non-ideal regenerator and found expressions for the optimal absorber temperature which led to the maximized power output of the Stirling engine. They evaluated the corresponding thermal efficiency to the maximized power and studied effect of variations in engine parameters on the obtained values of the maximized power and its corresponding thermal efficiency [17]. They also optimized the solar-dish Stirling engine mathematically in a single objective optimization approach with only one design parameter (decision variable).

This paper was an extension of the work performed by Yaqi et al. [17] with simultaneous consideration of two objective functions instead of the single objective function. In this regard, two objective functions including output power and thermal efficiency of the entire solar-dish Stirling system were simultaneously considered. Furthermore, multi-objective optimization was conducted with three decision variables including solar absorber temperature and the highest and lowest temperatures of the working fluid in the heat engine.

Solving multi-objective optimization problems is a very difficult goal which requires simultaneous satisfaction of a number of different and even conflicting objectives. Evolutionary algorithms were initially extended and applied during the mid-eighties in an attempt to stochastically solve problems of this generic class [35]. A reasonable solution to a multi-objective problem is to investigate a set of solutions, each of which satisfies the objectives at an acceptable level without being dominated by any other solution [36]. Multi-objective optimization problems generally show a possibly uncountable set of solutions, namely Pareto frontier, the evaluated vectors of which represent the best possible trade-offs in the objective function space. During this term, multi-objective optimization of different thermodynamic and energy systems has been considered by researchers nowadays [16,37–42].

In this paper, a thermodynamic model was developed for evaluating the output power and thermal efficiency of a solar-dish Stirling engine. The developed model considered convective and

radiative heat transfers at the hot end of the engine as well as the convective heat transfer at the cold end of the engine. Moreover, non-ideal regeneration performance and conductive thermal bridge loss between the heat source and heat sink were taken into account. The cyclic time of the engine was evaluated based on the time spent on the compression, expansion and regeneration processes. This model was applied to evaluate the output power and thermal efficiency of the entire solar-dish Stirling system for the multi-objective optimization process. Considering three design parameters (decision variables) including the absorber temperature and the highest and lowest temperatures of the working fluid in the engine, aforementioned objective functions were optimized (maximized) using the multi-objective evolutionary algorithm (MOEA) and the Pareto frontier was obtained in the objective space (power and efficiency space). One class of efficient MOEA, namely non-dominated sorting genetic algorithm i.e. NSGA-II, was employed for maximizing the output power and thermal efficiency simultaneously. A final optimal solution from available solutions located on the Pareto frontier was selected using three decision-making approaches including the fuzzy Bellman–Zadeh, LINMAP and TOPSIS methods. The final solution obtained by each decision maker was compared with the corresponding optimal solutions introduced by other decision making tools and the optimal solution introduced in the single objective optimization as shown in [17]. The superiority of solutions obtained using the multi-objective approach over the corresponding results obtained by the conventional approach ([17]) was discussed and emphasized. Sensitivity of the results against variation of the solar-dish Stirling engine parameters including the solar collector concentration ratio, effectiveness of the regenerator, conductive thermal bridge loss coefficient, volumetric ratio and solar flux intensity was also studied in detail.

2. Solar-dish Stirling system

In solar-dish Stirling systems, a mirror of the parabolic shaped concentrator focuses solar light on the focal point of the concentrator where hot end of the Stirling engine is installed. Therefore, solar energy with a relatively high temperature is transferred to the hot

side heat exchanger of the Stirling engine. Fig. 1 illustrates a schematic for a solar-dish Stirling engine connected to a solar dish concentrator. The solar-dish is equipped with a sun tracker which tracks the sun in order to have the maximum solar energy transfer to the engine when the sun moves during the days. Hence, the maximum solar energy is absorbed and transferred to the working fluid in the hot space of the engine.

As shown in Fig. 2, ideal Stirling cycle consists of four processes including two isothermal (1–2 and 3–4) and two isochoric processes (2–3 and 4–1) in the regenerator. In a real cycle, it is impractical to have an ideal heat transfer in the regenerator, in which the entire amount of absorbed heat (in the process 4–1) is transferred to the working fluid in the isochoric heating process (process 2–3). Therefore, a heat transfer loss occurs in the regenerator. In addition, a conductive heat transfer occurs between the heat source and sink, namely conductive thermal bridge loss (Q_0).

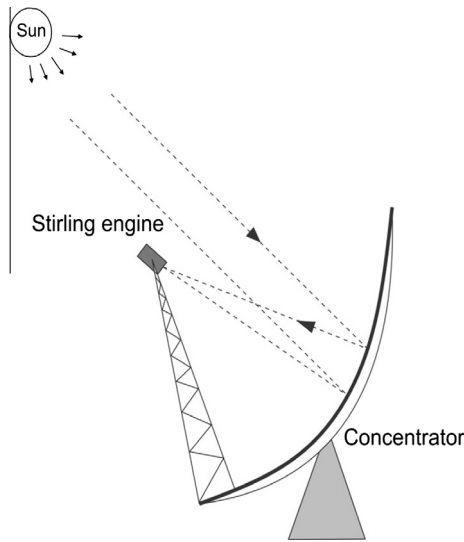


Fig. 1. Schematic of a solar Stirling engine.

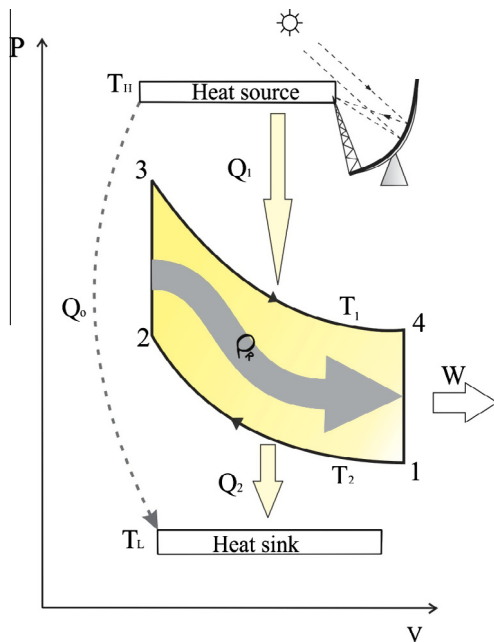


Fig. 2. P–V diagram of an isothermal solar Stirling engine cycle.

3. Thermodynamic modeling of the solar-dish Stirling engine

The actual useful heat gain of the dish collector, considering conduction, convection and radiation losses, is given as [13]:

$$q_u = IA_{app}\eta_0 - A_{rec} \left[h(T_H - T_0) + \varepsilon\delta(T_H^4 - T_0^4) \right] \quad (1)$$

In Eq. (1), I is the direct solar flux intensity, η_0 is the collector optical efficiency, A_{rec} and A_{app} are the absorber and collector aperture areas, respectively, h is the conduction/convection heat transfer coefficient in the solar dish, T_H and T_0 are the absorber and ambient temperatures, respectively, ε is the emissivity factor of the collector and δ is the Stefan–Boltzman constant.

Thermal efficiency η_s of the dish collector is obtained as follows [17]:

$$\eta_s = \frac{q_u}{IA_{app}} = \eta_0 - \frac{1}{IC} [h(T_H - T_0) + \varepsilon\delta(T_H^4 - T_0^4)] \quad (2)$$

It is important to mention that there is a finite heat transfer rate in the regenerator, in which the magnitude (Q_R) is given by the following expression [17,18,29,30,34]:

$$Q_R = nC_V\varepsilon_R(T_1 - T_2) \quad (3)$$

where C_V is molar specific heat of the working fluid in the regenerative processes, ε_R is effectiveness of the regenerator and n is molar mass of the working fluid. Thus, the regenerative heat loss in two regenerative processes is given as follows [17,18,29,30,37]:

$$\Delta Q_R = nC_V(1 - \varepsilon_R)(T_1 - T_2) \quad (4)$$

Owing to the influence of irreversibility of the finite-rate heat transfer, time of the regenerative processes is not negligible in comparison to that of two isothermal processes [29]. In order to calculate time of the regenerative processes, it is assumed that temperature of the working fluid in the regenerative processes, as a function of time, is given as follows [17–19,30,34]:

$$\frac{dT}{dt} = \pm M_i \quad (5)$$

where M is proportionality constant which is independent from the temperature difference and depends only on the property of regenerative material, called regenerative time constant, and $\pm$ sign belongs to the heating ($i=1$) and cooling ($i=2$) processes, respectively [17]. Therefore:

$$t_3 = \frac{T_1 - T_2}{M_1} \quad (6)$$

$$t_4 = \frac{T_1 - T_2}{M_2} \quad (7)$$

Total regenerative time is obtained as follows:

$$t_{re} = t_3 + t_4 \quad (8)$$

For a Stirling cycle, the amounts of heat released to the absorber and absorbed by the heat sink are as follows [17]:

$$Q_1 = [h_{HC}(T_H - T_1) + h_{HR}(T_H^4 - T_1^4)]t_1 = nRT_1 \ln \lambda + nC_V(1 - \varepsilon_R)(T_1 - T_2) \quad (9)$$

$$Q_2 = h_{LC}(T_2 - T_L)t_2 = nRT_2 \ln \lambda + nC_V(1 - \varepsilon_R)(T_1 - T_2) \quad (10)$$

where Q_1 is amount of the heat absorbed by the working fluid at the temperature/from the absorber at temperature T_H . Similarly, Q_2 is amount of the heat released by the working fluid at the temperature/to the heat sink at temperature T_L . In Eq. (9), h_{HC} and h_{HR} are convection and radiative heat transfer coefficients at the hot end of the engine, respectively. In Eq. (10), h_{LC} is the convective heat transfer coefficient on the cold side heat exchanger. Furthermore, in Eqs. (9) and (10), R is the universal gas constant and λ is

volumetric ratio of the engine which is equal to the ratio of volumes during the regenerative processes, i.e.

$$\lambda = \frac{V_1}{V_2} \quad (11)$$

In addition, there is heat leakage directly from the heat source to the heat sink which is directly proportional to the temperature difference of sources and the cycle time; i.e. [17,18,29,30],

$$Q_0 = k_0(T_H - T_L)t \quad (12)$$

where k_0 is conductive thermal bridge loss coefficient, t is cyclic period of the temperatures, T_H and T_L are temperatures of the absorber and heat sink, respectively.

The net heat absorbed Q_H from the heat source and heat released Q_L to the heat sink are obtained using the following expressions:

$$Q_H = Q_1 + Q_0 \quad (13)$$

$$Q_L = Q_2 + Q_0 \quad (14)$$

Using Eqs. (6)–(11), it is obtained that the cyclic period t is:

$$t = t_1 + t_2 + t_{re} \quad (15)$$

$$= \frac{nRT_1 \ln \lambda + nC_v(1 - \varepsilon_R)(T_1 - T_2)}{h_{HC}(T_H - T_1) + h_{HR}(T_H^4 - T_1^4)} + \frac{nRT_2 \ln \lambda + nC_v(1 - \varepsilon_R)(T_1 - T_2)}{h_{LC}(T_2 - T_L)} + \left(\frac{1}{M_1} + \frac{1}{M_2} \right) (T_1 - T_2)$$

Considering cyclic period of the Stirling engine, the power output and thermal efficiency of the engine are given as follows:

$$p = \frac{W}{t} = \frac{Q_H - Q_L}{t} \quad (16)$$

$$\eta_t = \frac{Q_H - Q_L}{Q_H} \quad (17)$$

Substituting Eq. (15) in Eqs. (16) and (17) leads to the following expressions for power and thermal efficiency of the Stirling engine:

$$p = \frac{T_1 - T_2}{\frac{T_1 + M(T_1 - T_2)}{h_{HC}(T_H - T_1) + h_{HR}(T_H^4 - T_1^4)} + \frac{T_2 + M(T_1 - T_2)}{h_{LC}(T_2 - T_L)} + F_1(T_1 - T_2)} \quad (18)$$

$$\eta_t = \frac{T_1 - T_2}{T_1 + M(T_1 - T_2) + [k_0(T_H - T_L)] \left[\frac{T_1 + M(T_1 - T_2)}{h_{HC}(T_H - T_1) + h_{HR}(T_H^4 - T_1^4)} + \frac{T_2 + M(T_1 - T_2)}{h_{LC}(T_2 - T_L)} + F_1(T_1 - T_2) \right]} \quad (19)$$

where

$$M = \frac{C_v(1 - \varepsilon_R)}{R \ln \lambda} \quad (20a)$$

$$F_1 = \frac{1}{nR \ln \lambda} \left(\frac{1}{M_1} + \frac{1}{M_2} \right) \quad (20b)$$

Thermal efficiency of the system is a product of thermal efficiency of the collector (Eq. (2)) and optimal thermal efficiency of the Stirling engine (Eq. (19)) [17]. Therefore:

$$\eta_m = \eta_s \eta_t \quad (21a)$$

$$\eta_m = \left\{ \eta_0 - \frac{1}{\varepsilon_C} \left[h(T_H - T_0) + \varepsilon \delta (T_H^4 - T_0^4) \right] \right\}$$

$$\left\{ \frac{1-x}{1+M(1-x)+[k_0(T_H-T_L)] \left[\left(\frac{1+M(1-x)}{h_{HC}(T_H-T_{1opt})+h_{HR}(T_H^4-T_{1opt}^4)} + \frac{x+M(1-x)}{h_{LC}(xT_{1opt}-T_L)} \right) (1+b) \right]} \right\} \quad (21b)$$

4. Multi-objective optimization

4.1. Evolutionary algorithms

Genetic algorithms were developed by John Holland in the 1960s as a means of importing mechanisms of natural adaptation into computer algorithms and numerical optimization [44]. Genetic algorithms apply an iterative, stochastic search strategy to find an optimal solution and imitate principles of biological evolution in a simplified manner (Fig. 3). In this study, the Pareto frontier was obtained using Genetic Algorithm (GA) as a branch of evolutionary algorithm. This method is a powerful optimization tool for nonlinear problems [35,37]. Pareto optimality is the key concept for establishing a hierarchy among the solutions of a multi-objective optimization problem in order to determine whether a solution is really one of the best possible trade-offs [37]. NSGA-II algorithm was utilized here as a multi-objective optimization method in order to find the Pareto frontier using genetic algorithm. Details of NSGA-II were given in [45].

4.2. Objective functions, decision variables and constraints

Two objective functions of this study included output power of the system (P) and the system thermal efficiency (η_m) denoted by Eqs. (18) and (21b), respectively.

In this paper, three decision variables were considered as follows:

T_1 : highest temperature of the working fluid at isothermal process (process 3–4 in Fig. 1).

T_2 : lowest temperature of the working fluid at isothermal process (process 1–2 in Fig. 1).

T_H : absorber temperature.

It is required to mention that the number of decision variables could be increased to four decision variables if volumetric ratio of the engine (λ) is considered an additional decision variable. However, in this research, it was required to have the highest possible value of λ in order to have the highest values for the maximum values of the output power and thermal efficiency. In other words, λ is always selected at its upper bound values defined for a variation of this variable. Due to the engineering limitation, λ is usually chosen from a range between 2.0 and 5.0. Therefore, if this constraint is implemented for λ , optimization leads to $\lambda = 5.0$ for this variable. Thus, the optimization would be meaningless for this variable and it is selected at its highest possible value. Hence, in this paper, it was removed from the list of decision variables.

The objective functions were solved with respect to the following constraints:

$$T_1 > T_2 \quad (22)$$

$$T_2 > 320^\circ\text{C} \quad (23)$$

$$700^\circ\text{C} \leq T_H \leq 1600^\circ\text{C} \quad (24)$$

$$0.4 \leq T_2/T_1 \leq 0.7 \quad (25)$$

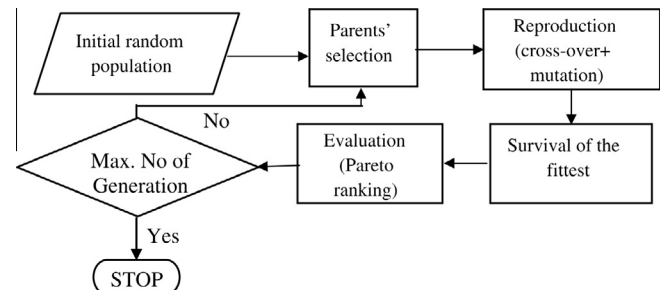


Fig. 3. Schematic of the objective space.

5. Decision-making in multi-objective optimization

In multi-objective optimization, a process of decision-making is required for selection of the final optimal solution from available solutions. There are several methods for decision-making process in decision problems. These methods can be employed for selection of a final optimal solution from the Pareto frontier. Since dimension of various objectives in a multi-objective optimization problem might be different (for example, in the present case, thermal efficiency objective had no dimension while dimension of the output power was in Watt), before any decisions, dimensions and scales of objective space should be unified. In this regard, objective vectors should be non-dimensioned before the decision-making process. There are several methods of non-dimensioning which are utilized in decision making and include linear non-dimensioning, Euclidean non-dimensioning and fuzzy non-dimensioning. In this fuzzy Bellman–Zadeh, LINMAP and TOPSIS methods of decision making methods were employed and the Bellman–Zadeh method utilized fuzzy non-dimensioning while other methods (LINMAP and TOPSIS) employed Euclidean non-dimensioning. Details of fuzzy non-dimensioning and Euclidean non-dimensioning are given in [43].

5.1. Bellman–Zadeh decision-making method

A final decision is defined by the Bellman and Zadeh model as the intersection of all fuzzy criteria and constraints and is represented by its membership function. In this method, a matrix of membership function is defined. The column of this matrix includes fuzzy membership function for each objective. Rows of the membership matrix reflect magnitudes of the membership functions for each solution of the Pareto frontier. Therefore, the number of columns in the membership function matrix is equal to the number of objectives and the number of rows is equal to the number of solutions located on the Pareto frontier. Details of the method of defining the membership function were presented by Sayyaadi and Mehrabipour [43]. The fuzzy membership function for each solution is set at the minimum value of the membership functions of all objectives for the proposed solution. Hence, a vector of membership is obtained, in which the elements are the minimum membership function of objectives at each solution. Finally, maximum of these minimums, i.e. an element of the membership vector with maximum values of membership function, is selected as a final solution. In other words, in the Bellman–Zadeh fuzzy decision making, a maximum of minimums (minimum membership function of objectives) is selected as the final optimal solution. Readers are encouraged to refer to [43] for more details.

5.2. LINMAP decision-making method

An ideal point on the Pareto frontier is a point at which each objective is optimized regardless of the satisfaction of other objectives. It is clear that, in the multi-objective optimization, it is impossible to have each objective in its optimal condition which can be obtained in a single objective optimization. Therefore, the ideal point is not located on the Pareto frontier. In the LINMAP method, after Euclidean non-dimensionalization of all objectives, the spatial deviation of each solution on the Pareto frontier from the ideal point is determined. Finally, a solution with a minimum spatial distance from the ideal point is selected as a desired final optimal solution. More details of the LINMAP method were presented by Sayyaadi and Mehrabipour [43].

5.3. TOPSIS decision-making method

In this method, beside the ideal point, a “non-ideal point” is defined. The non-ideal point is the ordinate in objective space, in

which each objective has its worst value. Therefore, beside the solution spatial distance from the ideal point, the solution spatial distance from the non-ideal one is used as a criterion for selecting the final solution. In the TOPSIS method, a solution with the maximum distance from the non-ideal point and minimum distance from the ideal point is selected as a desired final optimal solution. More details of the TOPSIS method could be found in [43].

6. Result and discussion

Output power and thermal efficiency of the solar-dish Stirling engine were maximized simultaneously using the multi-objective optimizing method, which worked based on the NSGA-II algorithm. In this regard, optimization was performed by objective functions, as expressed by Eqs. (18) and (21b), and constraints, as stated in Eqs. (22)–(25). Design parameters (decision variables) of the optimization were temperature of the working fluid in the high temperature isothermal process (T_1), temperature of the working fluid in the low temperature isothermal process (T_2) and the absorber temperature (T_H).

To be consistent with the previous works, specifications of the solar-dish Stirling engine were considered as follows [17]:

$$\begin{aligned} x &= 0.5, h_{HC} = h_{LC} = 200 \text{ W K}^{-1}, h_{HR} = 4 \times 10^{-8} \text{ W K}^{-4}, n \\ &= 1 \text{ mol}, \lambda = 2, \varepsilon_R = 0.90, \varepsilon = 0.90, R = 4.3 \text{ J mol}^{-1} \text{ K}^{-1}, C_v \\ &= 15 \text{ J mol}^{-1} \text{ K}^{-1}, \delta = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}, T_L = 320 \text{ K}, T_0 \\ &= 300 \text{ K}, h = 20 \text{ W m}^{-2} \text{ K}^{-1}, T_H = 1100 \text{ K}, I = 1000 \text{ W m}^{-2}, C \\ &= 1300, \left(\frac{1}{M_1} + \frac{1}{M_2} \right) = 2 \times 10^{-5} \text{ s K}^{-1} \end{aligned}$$

Fig. 4 illustrates the Pareto frontier in the proposed objectives' space obtained by the multi-objective evolutionary algorithm (NSGA-II). In this figure, the ideal and non-ideal are specified. Furthermore, three final solutions selected by the fuzzy, LINMAP and TOPSIS decision makers are indicated in Fig. 4.

Table 1 compares optimal results obtained in this paper using three decision making methods with the corresponding results obtained in a single objective-single decision variable work [13]. Furthermore, this table compares solutions with the ideal solution (ideal solution is illustrated in Fig. 4). It is clear that thermal efficiency of the final solutions selected using the TOPSIS, LINMAP and fuzzy decision-makers was 4.87%, 4.13% and 1.23% less than the corresponding thermal efficiency obtained in [17], respectively. Thermal efficiency of the solutions obtained using the TOPSIS, LINMAP and fuzzy decision makers was 8.06%, 7.32% and 4.42% less

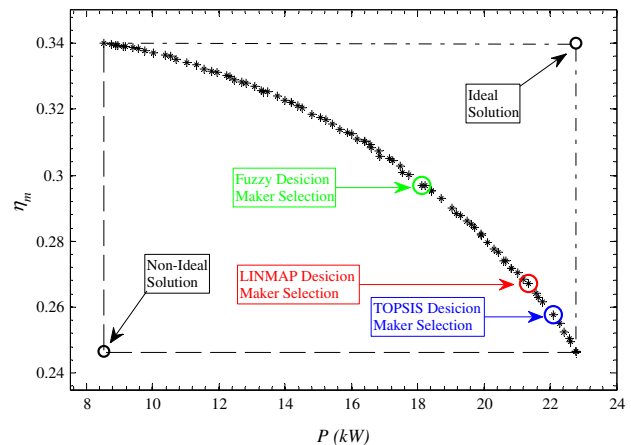


Fig. 4. Pareto optimal frontier in the objective space.

Table 1

Comparison of the final optimal solutions of this paper (multi-objective, multi-variable) with the corresponding results obtained in [17] (single objective–single variable optimization).

Optimal solution	Decision making method	Decision variable			Objectives		Deviation index from the ideal solution d
		T_1 (K)	T_2 (K)	T_H (K)	P (W)	η_m	
This work	TOPSIS	1248.3	571.4	1565.6	22286.8	0.2594	0.0682
	LINMAP	1248.0	539.4	1569.2	21587.4	0.2668	0.0650
	FUZZY	1192.7	506.8	1449.5	18113.8	0.2958	0.0786
Ref. [13]	–	923.9	462.0 ^a	1100.0 ^a	10164.2	0.3081	0.2252
Ideal solution	–	–	–	–	22800.0	0.3400	0.0000

^a This parameter was not considered a decision variable in Ref. [17]. Its fix value in [17] is indicated in the table.

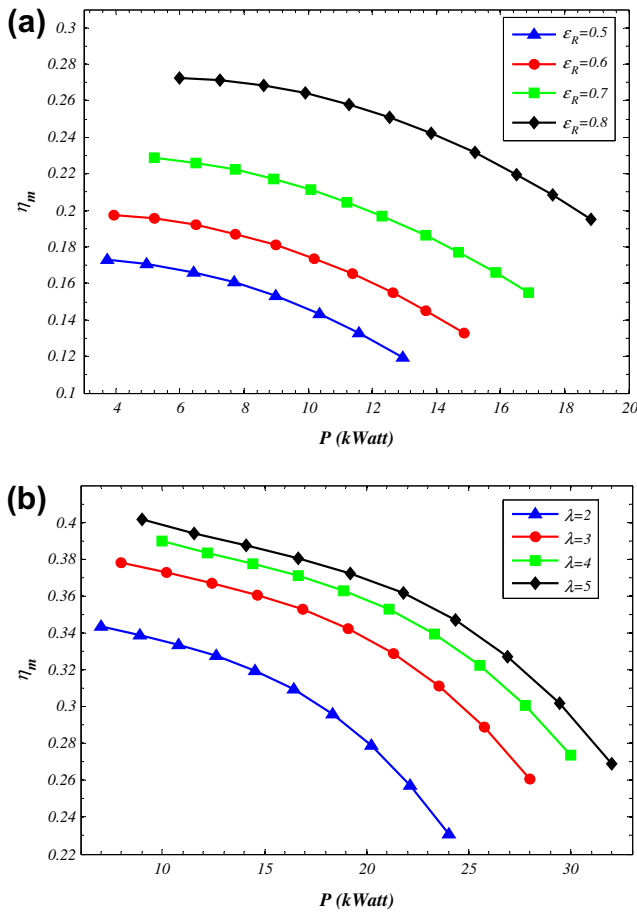


Fig. 5. Pareto optimal frontier in objective space at various (a) degrees of effectiveness of regenerator; (b) volumetric ratio of the engine.

than that of the ideal solution, respectively. This table indicates that the best value for the thermal efficiency obtained in this paper was 1.23% less than that of [17] and 4.42% less than that of the ideal solution. Comparison of the output power of the engine obtained using three decision making with the corresponding output power obtained in [17] indicated that output power of the solutions obtained by TOPSIS, LINMAP and fuzzy methods was 119.3%, 112.4% and 78.2% higher than the output power obtained in [17]. The output power obtained in this paper for three solutions using the TOPSIS, LINMAP and fuzzy decision making methods was 2.2%, 5.3% and 20.6% less than the output power of the ideal solution, respectively. Results in Table 1 indicate that, although thermal efficiency of solutions obtained in this work was less than the corresponding thermal efficiency obtained in the single objec-

Table 2a

Optimal solutions of the Stirling engine at various volumetric ratios of the engine ($\epsilon_R = 0.90$).

λ	Decision making method	Decision variable			Objectives	
		T_1 (K)	T_2 (K)	T_H (K)	P (W)	η_m
2.0	TOPSIS	1248.3	571.4	1565.6	22286.8	0.2594
	LINMAP	1248.0	539.4	1569.2	21587.4	0.2668
	FUZZY	1192.7	506.8	1449.5	18113.8	0.2958
3.0	TOPSIS	1250.5	539.8	1589.9	26356.8	0.2865
	LINMAP	1250.5	539.8	1589.9	26356.8	0.2865
	FUZZY	1208.4	492.6	1465.2	21410.8	0.3279
4.0	TOPSIS	1275.1	544.9	1593.0	28726.0	0.2976
	LINMAP	1273.3	538.4	1570.4	27832.5	0.3062
	FUZZY	1226.7	509.9	1436.7	22312.1	0.3427
5.0	TOPSIS	1261.3	534.5	1575.2	29002.6	0.3102
	LINMAP	1261.6	534.7	1548.9	28133.4	0.3178
	FUZZY	1216.8	498.2	1443.1	23254.3	0.3509

tive work [17], much greater enhancement was obtained for the output power objective in comparison to [17]. In other words however, the least thermal efficiency of the Stirling engine obtained in this work (using TOPSIS decision maker) was 4.87% less than the corresponding thermal efficiency of the single objective–single variable work [17] and the least output power (obtained by the fuzzy decision maker) was 78.2% higher than the corresponding output power of [17]. For better insight into the status of various solutions obtained here in comparison to the solution of the single objective–single variable work [17], a deviation index characterizing deviation of each solution from the ideal solution was also introduced.

$$d_+ = \sqrt{(p_n - p_{n,ideal})^2 + (\eta_{m_n} - \eta_{m_n,ideal})^2}. \quad (26)$$

$$d_- = \sqrt{(p_n - p_{n,non-ideal})^2 + (\eta_{m_n} - \eta_{m_n,non-ideal})^2} \quad (27)$$

$$d = \frac{d_+}{(d_+) + (d_-)} \quad (28)$$

p_n and η_{m_n} denote Euclidian non-dimensioned output power and thermal efficiency (indexes *ideal* and *non-ideal* denote the corresponding values of objective functions at the ideal and non-ideal points, respectively). The last column of Table 1 denotes deviation index (d) for the solution of each optimization approach. As is clear, deviation indexes for the multi-objective optimization approach of this paper were 0.0682, 0.0650 and 0.0786 for the solutions obtained by TOPSIS, LINMAP and fuzzy decision making methods, respectively. These values were comparable with the corresponding value obtained for the single objective–single variable optimization [17], where deviation index is 0.2252. This issue implies that deviation indexes of the multi-objective, multi-variable optimization for all decision making methods were much lower than the single objective–single variable optimization (the ideal deviation index was zero). Also, the last column of Table 1 indicates that the LIN-

Table 2b

Optimal solutions of the Stirling engine at various degrees of effectiveness of the regenerator ($\lambda = 2.0$) parameter.

ε_R	Decision making method	Decision variables			Objectives	
		T_1 (K)	T_2 (K)	T_H (K)	P (W)	η_m
0.5	TOPSIS	1195.5	692.2	1535.6	12132.6	0.1289
	LINMAP	1199.7	634.4	1539.6	11536.1	0.1348
	FUZZY	1154.1	582.6	1411.0	9376.0	0.1507
0.6	TOPSIS	1222.9	655.8	1575.6	13845.9	0.1453
	LINMAP	1218.2	651.3	1557.6	13556.8	0.1480
	FUZZY	1169.4	575.3	1420.4	10649.2	0.1706
0.7	TOPSIS	1240.8	674.7	1555.7	15853.0	0.1671
	LINMAP	1220.7	628.9	1543.2	15153.9	0.1745
	FUZZY	1167.8	539.4	1453.6	12262.3	0.1965
0.8	TOPSIS	1243.3	601.2	1577.3	18281.2	0.2047
	LINMAP	1236.7	585.0	1529.4	17112.3	0.2164
	FUZZY	1189.6	527.6	1446.1	14422.1	0.2372
0.9	TOPSIS	1248.3	571.4	1565.6	22286.8	0.2594
	LINMAP	1248.0	539.4	1569.2	21587.4	0.2668
	FUZZY	1192.7	506.8	1449.5	18113.8	0.2958

MAP decision making method led to a lower value for deviation index; hence, in this paper, the solution that was selected using the LINMAP decision making was considered a final desired optimal solution of the multi-objective, multi-variable optimization for the solar-dish Stirling engine. It should be noted that this is not a general rule; as in other cases, other decision making methods may yield better results.

Fig. 5a and b shows the Pareto optimal frontier at various degrees of effectiveness of the regenerator and volumetric ratios of the engine, respectively.

Optimal solutions introduced by three decision makers in the cases that the engine was designed with different volumetric ratios are indicated in Table 2a. Similar results at various degrees of effectiveness of the regenerator are indicated in Table 2b. As expected, engines with higher values of volumetric ratio and regenerator effectiveness had higher optimal values for both objectives (power and thermal efficiency).

Fig. 6a–d illustrate absorber temperature of the Stirling engine for solutions of the Pareto frontier at various degrees of effectiveness of the regenerator (ε_R) and volumetric ratio (λ). Fig. 6a and b shows that, for solutions of the Pareto frontier with higher values of thermal efficiency, the absorber temperature (T_H) was lower. As previously mentioned, all the solutions located on the Pareto frontier could be considered a potential final optimal solution of the multi-objective optimization. In this regard, the solutions located on the left side of the Pareto frontier (Fig. 4) were solutions with higher thermal efficiency and lower output power. It means that, if a weight factor were considered for each objective (a value of ≤ 1.0), solutions located on the right side of the Pareto frontier would have higher weight factors for the thermal efficiency and lower weight factors for the output power. Fig. 6a and b imply that, for solutions of the Pareto frontier with a higher weight factor for thermal efficiency, values of absorber temperature were lower. When the weight factor for thermal efficiency was 1.0 (the first solution on the left side of the Pareto frontier), the absorber temperature was 1100 K, which was consistent with the results ob-

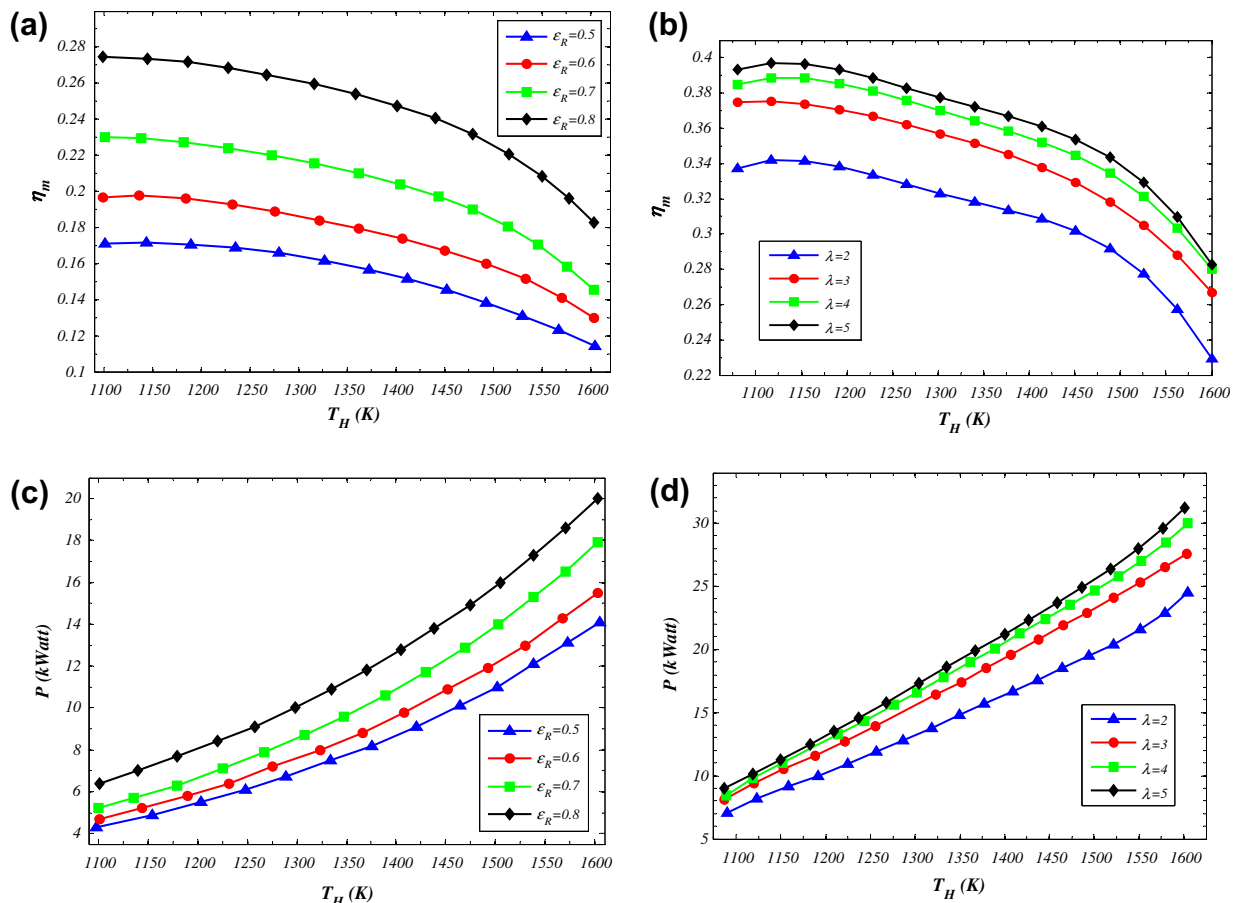


Fig. 6. Optimal solutions of the Pareto frontier as a function of the absorber temperature (T_H), effectiveness of the regenerator (ε_R) and volumetric ratio of the engine (λ) (a) $\eta_m - T_H - \varepsilon_R$; (b) $\eta_m - T_H - \lambda$; (c) $P - T_H - \varepsilon_R$; (d) $P - T_H - \lambda$.

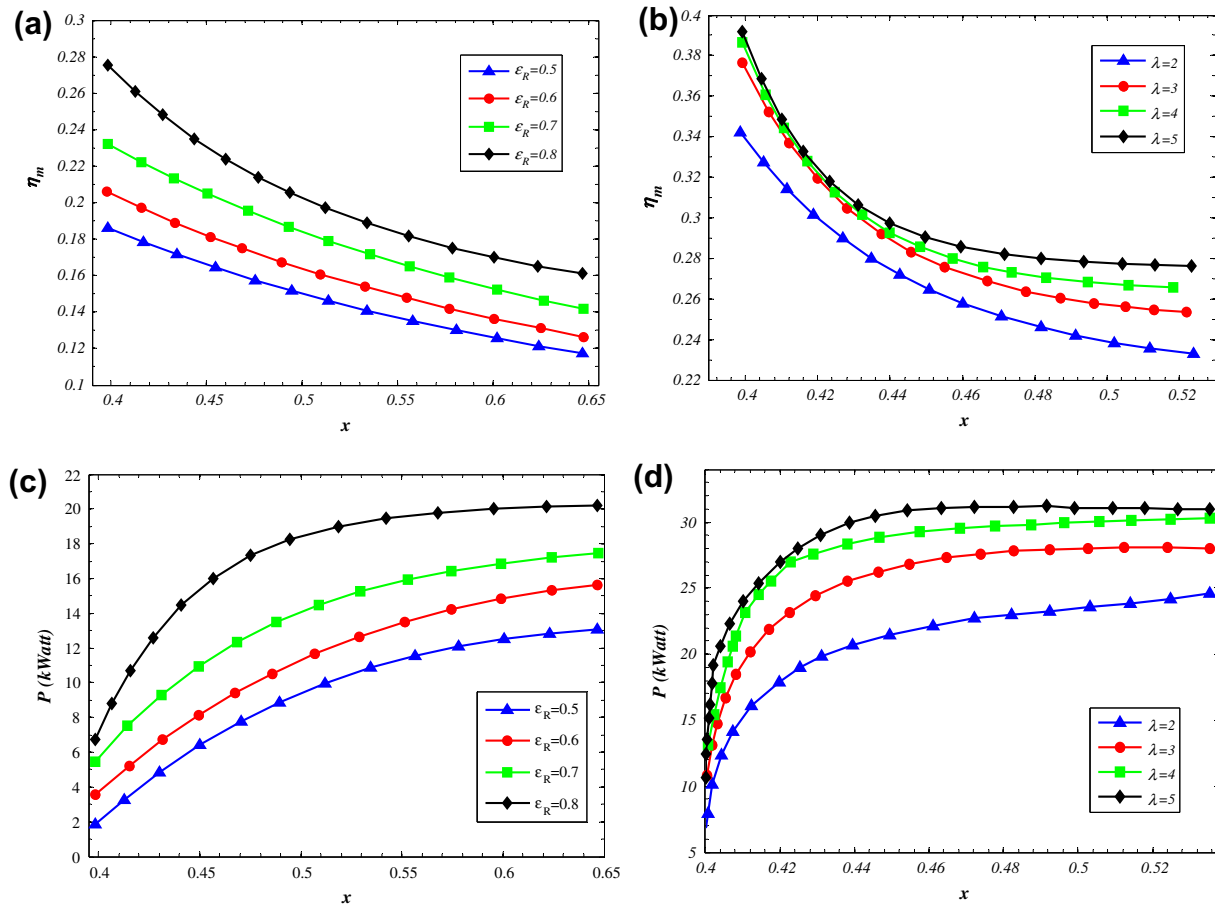


Fig. 7. Optimal solutions of the Pareto as a function of the temperature ratio of the engine (x), effectiveness of the regenerator (ϵ_R) and volumetric ratio of the engine (λ) (a) $\eta_m - x - \epsilon_R$; (b) $\eta_m - x - \lambda$; (c) $P - x - \epsilon_R$; (d) $P - x - \lambda$.

Table 3
Sensitivity of objective functions and decision variables to variations of the system parameter.

Parameters	Proposed value	Possible variations		Power (kW)		Efficiency		T_1 (K)		T_2 (K)		T_H (K)	
		Lower limit	Upper limit	a	b	a	b	a	b	a	b	a	b
λ	2	2	5	17.83	23.09	0.30	0.35	1225.1	1212.9	500.1	486.1	1454.7	1449.2
ϵ_R	0.9	0.5	0.9	9.24	17.25	0.15	0.30	1170.3	1196.2	558.7	515.1	1437.4	1410.2
I (W m ⁻²)	1000	500	1500	9.24	18.52	0.15	0.32	1203.9	1216.2	567.4	501.5	1433.3	1479.0
C	1300	700	2000	16.63	19.18	0.25	0.32	1134.6	1231.8	513.9	514.7	1370.7	1491.8
k_0 (W K ⁻¹)	2.5	1.5	5	17.52	18.72	0.31	0.28	1220.5	1227.1	503.1	506.6	1438.5	1482.2

^a Value corresponds to the lower limit of the proposed parameter.

^b Value corresponds to the upper limit of the proposed parameter.

Table 4
Error analysis based on the mean absolute percent error (MAPE) method.

Error analysis	FUZZY		LINMAP		TOPSIS	
	P	η_m	P	η_m	P	η_m
Maximum error (%)	2.02	3.65	1.71	4.37	2.70	3.82
Average error (%)	0.64	1.65	1.14	1.90	1.33	1.54

tained in [13]. The absorber temperature increased to 1600 K when the weight factor for thermal efficiency decreased to 0.0; i.e. weight factor for the output power was considered 1.0 (the first solution on the right side of the Pareto frontier). Therefore, the absorber temperature of a final solution with greater weight for the output power should be kept at a higher value than the corre-

sponding absorber temperature of solutions with higher weight factor for the thermal efficiency. This fact is also elucidated in Fig. 6c and d, which present the output power of the Pareto solutions as a function of the absorber temperature. In addition, Fig. 6a–d demonstrate that output power and thermal efficiency increase with the increase in effectiveness of the regenerator and volumetric ratio of the engine.

Fig. 7a–d represent similar graphs for solutions of the Pareto frontier. These figures illustrate the magnitude of objectives for solutions of the Pareto frontier as functions of the temperature ratio (x), effectiveness of the regenerator (ϵ_R) and volumetric ratio of the engine (λ). They also show that, for solutions with higher values of weight factors for thermal efficiency, temperature ratio of the engine was lower than the corresponding temperature ratio of the solutions with lower weight factors for thermal efficiency. Hence, when a higher weight factor was considered for the output

power, the engine should be designed with a higher temperature ratio.

6.1. Sensitivity analysis

Optimization is usually performed while some parameters are considered to be fixed. In sensitivity analysis, effect of variations in these kinds of parameters on the final selected solutions is studied. Sensitivity analysis was performed here in order to evaluate effect of variations in system parameters including volumetric ratio, effectiveness of the regenerator, solar flux intensity, concentration ratio of the solar dish and conductive thermal bridge loss coefficient. Table 3 indicates effect of variations in aforementioned parameters of objective functions and decision variables.

This table implies high sensitivity of objective functions against variations of the proposed system parameters. Therefore, in evaluation of the parameters including volumetric ratio, effectiveness of the regenerator, solar flux intensity, concentration ratio of the solar dish and conductive thermal bridge loss coefficient, special care must be paid and results obtained in the condition in which these parameters are evaluated should be generalized to other cases with caution.

6.2. Error analysis

In this paper, error analysis was performed based on the mean absolute percent error (MAPE) method. In this regard, 30 runs for the computer code were performed and a final solution for each run was obtained using the fuzzy, LINMAP and TOPSIS decision making methods. Then, magnitudes of each objective (output power and thermal efficiency) were compared with its corresponding best value (presented as a final solution in this paper). The first row of this table represents average error of each objective for solutions of three decision making methods. The last row of this table indicates a maximum error of each objective obtained using three decision making methods.

According to the data in Table 4, it can be found that average error of the solutions obtained by three decision making methods were 1.04% and 1.70% of the output power and thermal efficiency, respectively. Similarly, it can be found that the maximum error of solutions obtained by three decision making methods were 4.37% and 2.02% of the output power and thermal efficiency, respectively.

7. Conclusions

A thermodynamic model was applied to determine output power and thermal efficiency of the solar-dish Stirling heat engine. The output power and thermal efficiency of the engine were considered simultaneously for multi-objective optimization, in which temperature of the working fluid in the high temperature isothermal process, temperature of the working fluid in the lower temperature isothermal process and the absorber temperature were considered design parameters. A final optimal solution was selected from available solutions of the Pareto frontier using three decision making methods including LINMAP, TOPSIS and fuzzy methods. It was shown that the multi-objective, multi-variable approach could lead to a more desirable design of the Stirling cycle if compared with the single objective-single variable approach performed in the previous researches. The priority of the multi-objective, multi-variable approach over the traditional single objective-single variable one was proven in all cases and significant lower deviation indexes were obtained for the multi-objective, multi-variable approach. Moreover, in this case, it was observed that the LINMAP decision making method could yield a final solution with the least deviating index from the ideal solution. The volu-

metric ratio effect and effectiveness of the regenerator on the final selected solution was studied and it was found that a final optimal solution with higher values of objectives (thermal efficiency and output power) could be achieved if these parameters were increased. It was also shown that, in the multi-objective optimization of the solar-dish Stirling engine, if a great weight factor is considered for the thermal efficiency, absorber temperature and temperature ratio of the engine should be considered at a lower value in comparison to the case in which a lower weight factor is considered for the thermal efficiency. Relatively high sensitivity of the final selected optimal solution against variation of the system parameters including the volumetric ratio, effectiveness of the regenerator, solar flux intensity, concentration ratio of the solar dish and conductive thermal bridge loss coefficient was found; therefore, evaluation of these parameters in the design process of the solar-dish Stirling should be performed precisely.

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